Session: 2023-24	
Part A – Introduction	
Subject	Mathematics
Semester	II
Name of the Course	Algebra and Number Theory
Course Code	B23-MAT-201
Course Type: (CC/MCC/MDC/CC-M/DSEC/VOC/DSE/PC/AEC/VAC)	CC
Level of the course	100-199
Pre-requisite for the course (if any)	Mathematics as a subject at level 4.0 (Class XII)
Course Learning Outcomes(CLOs):	After completing this course, the learner will be able to: 1. Gain knowledge of the concepts of symmetric, skew-symmetric, Hermitian, skew-Hermitian, Orthogonal and Unitary matrices, Linear dependence and independence of rows and columns of a matrix. Have knowledge of procedure and cognitive skills used in calculating rank of a matrix, eigen values, characteristic equation, minimal polynomial of a matrix and technical skills used in solving problems based on Cayley- Hamilton theorem. 2. Have knowledge of the concepts used in solving problems based on relations between the roots and coefficients of general polynomial equation

	in one variable, solutions of polynomial equations having conditions on roots, common roots and multiple roots. Understand Descarte's rule of signs and learn cognitive and technical skills required in assessing nature of the roots of an equation and solving problems based on these.
	3. Have deeper and procedural knowledge required for solving cubic and biquadratic equations used in Mathematics as well as many other learning fields of study. To understand the basic concepts of number theory and their applications in problem solving and life- long learning.
	4. Have knowledge of concepts, facts, principles and theories of Linear Congruences, Fermat's theorem, Euler's theorem, Wilson's theorem and its converse, Chinese Remainder theorem. Attain cognitive skills used in solving linear Diophantine equations in two variables.
CLO 5 is related to the practical component of the course.	5. Attain cognitive and technical skills required to formulate and solve practical problems involving rank of a matrix, inverse of a matrix, Cardon's method, Ferrari's method, Descarte's method, Cayley-Hamilton theorem, Euler's theorem and Chinese Remainder theorem. Have technical and practical skills required for solving algebraic equations, finding inverse and eigen values of matrices by using built in functions of MAXIMA software.

Credits	Theory	Practical	Total
	3	1	4
Contact Hours	3	2	5
Internal Assessment Marks	20	10	30
End term Examination Marks	50	20	70
Examination Time	3 Hours	3 Hours	
Max. Marks:100			

Part B- Contents of the Course

Instructions for Paper- Setter

The examiner will set 9 questions asking two questions from each unit and one compulsory question by taking course learning outcomes (CLOs) into consideration. The compulsory question (Question No. 1) will contain 5 parts covering entire syllabus. The examinee will be required to attempt 5 questions, selecting one question from each unit and the compulsory question.

Unit	Topics	Contact Hours
I	Symmetric, Skew symmetric, Hermitian and skew Hermitian matrices, Elementary operations on matrices, Rank of a matrix, Inverse of a matrix, Linear dependence and independence of rows and columns of matrix, Row rank and column rank of a matrix, Eigen values, Eigen vectors and characteristic equation of a matrix, Minimal polynomial of a matrix, Cayley-Hamilton theorem and its use in finding the inverse of a matrix, Unitary and orthogonal matrices.	12
II	Relations between the roots and coefficients of general polynomial equation in one variable, Solutions of polynomial equations having conditions on roots, Common roots and multiple roots, Transformation of equations, Nature of the roots of an equation, Descarte's rule of signs.	12

III	Solutions of cubic equations (Cardon's method), Biquadratic equations and their solutions. Divisibility, Greatest common divisor (gcd), Least common multiple (lcm), Prime numbers, Fundamental theorem of arithmetic.	12
IV	Linear congruences, Fermat's theorem, Euler's theorem, Wilson's theorem and its converse, Chinese Remainder theorem, Linear Diophantine equations in two variables.	12
Practical		
	The practical component of the course has two parts, Problem Solving and Practical's using MAXIMA software. The examiner will set 4 questions at the time of practical examination asking two questions from the part (A) and two questions from the part (B) by taking course learning outcomes (CLOs) into consideration. The examinee will be required to solve one problem from the part (A) and to execute one problem successfully from the part (B). Equal weightage will be given to both the parts. The evaluation will be done on the basis of practical record, viva-voce, write up and execution of the program. **A) Problem Solving: Questions related to the following problems will be worked out and record of those will be maintained in the Practical Notebook:** 1. Problems to find the row rank and column rank of a matrix. 2. Problems to find the eigen values and eigen vectors of a matrix. 3. Problems to find the minimal polynomial of a matrix.	30

	4. Problems of finding inverse of a matrix using Cayley-Hamilton theorem.
	
	5. Problems of solving cubic equations by Cardon's method.
	
	6. Problems of solving biquadratic equations by Descarte's method.
	
	7. Problems of solving biquadratic equations by Ferrari's method.
	
	8. Problems to find gcd and lcm of two integers.
	
	9. Problems to find solution of linear congruence using Euler's theorem.
	
	10. Problems to find common solution of congruences using Chinese remainder theorem.
	
	B) **The following practicals will be done using MAXIMA Software and their record will be maintained in the practical note Book:**
	
	  1. To find roots of algebraic equations using MAXIMA.
	  2. To find multiple roots of algebraic equations using MAXIMA
	  3. To find the value of a determinant using MAXIMA.
	  4. To compute inverse of a square matrix using MAXIMA.
	  5. To find Eigen values of a square matrix using MAXIMA.
	  6. To find Eigen vectors of a square matrix using MAXIMA.
	  7. To solve system of linear equations using MAXIMA.
	  8. Problems to find gcd and lcm of two or more	

integers using MAXIMA. 9. Problems of solving biquadratic equations by Ferrari's method using MAXIMA.	

Suggested Evaluation Methods

Internal Assessment:	End Term Examination:
➢ **Theory** **20** • Class Participation: 5 • Seminar/presentation/assignment/quiz/class test etc.: 5 • Mid-Term Exam: 10 ➢ **Practicum** **10** • Class Participation: - • Seminar/Demonstration/Viva-voce/Lab records etc.: 10 • Mid-Term Exam: -	➢ **Theory** **50** Written Examination ➢ **Practicum** **20** Lab record, viva-voce, write up and execution of the program

Part C- Learning Resources

Recommended Books/e-resources:
1) Stephen H. Friedberg, Arnold J. Insel & Lawrence E. Spence (2022). *Linear Algebra* (5th edition). Prentice Hall of India Pvt. Ltd.
2) Seymour Lipschutz and Marc Lars Lipson (2013). *Linear Algebra*. (4th Edition) Schaum's Outline Series, McGraw-Hill.
3) K. B. Dutta (2004). *Matrix and Linear Algebra*. Prentice Hall of India Pvt. Ltd.
4) Vivek Sahai & Vikas Bist (2013). *Linear Algebra* (2nd edition). Narosa Publishing House.
5) I. Niven (1991). *An Introduction to the Theory of Numbers* (5th edition). John Wiley & Sons.
6) H.S. Hall and S.R. Knight (2023). *Higher Algebra* (7th edition). Arihant Publications.
7) Leonard Eugene Dickson (2009). *First Course in the Theory of Equations*. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785).